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MATHEMATICAL MODELING OF CONVECTIVE HEAT AND MASS TRANSFER IN THE DRYING OF SOLID PARTICLES IN A BED

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A closed system of equations is proposed for calculating convective heat and mass transfer in the drying of solid particles by a gaseous heat transfer agent in a moving bed. As an example, the operation of a belt-type dryer with crossed interaction of a drying agent and a bed of fruit cut into circular slices is considered. Results of a numerical solution of the problem are presented in figures.

The drying of solid particles in a bed by a gaseous heat transfer agent is widely used commercially. Interacting flows are most often arranged in a counterflow (column dryers) or a crossed-flow (belt conveyers with transverse blowing) scheme. The bed to be dried is a two-phase system of the type of moist solid-humid gas, where aerodynamic and heat and mass transfer processes must be described within the mechanics of heterogeneous systems. Substantial difficulties in the mathematical description are caused by the great diversity of the possible geometrical structure of the bed. Consequently, schematization of the structure of the bed and orientation to some averaged characteristics are inevitable.

The choice of averaging principles is an important step in the development of methods of the mechanics of heterogeneous media. Concepts of methods for space and time averaging are given in [1] and [2], respectively. The latter approach, which is more rigorous mathematically [3], will be used here. It is assumed in the description that the interacting media are nonviscous but they experience resistance in relative motion of the phases and the action of Archimedean forces (the hydraulic approach). Transfer of momentum and the kinetic component of the energy and dissipation are neglected. Only averaged parameters without the fluctuations are used. With these assumptions the equations of motion and energy have the form

$$\mathrm{B}_i + \mathrm{B}_j = 1\,,$$

$$\frac{\partial}{\partial t}(\rho_i \mathrm{B}_i W_i) = -\nabla\cdot(\rho_i \mathrm{B}_i W_i \mathbf{U}_i) - M_{ij} + M_{ji}\,,$$

$$\frac{\partial}{\partial t}[\rho_i \mathrm{B}_i (1 - W_i)] = -\nabla\cdot[\rho_i \mathrm{B}_i \mathbf{U}_i (1 - W_i)]\,, \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho_i \mathrm{B}_i \mathbf{U}_i) = -(\nabla\cdot\mathbf{U}_i)\,\rho_i \mathrm{B}_i \mathbf{U}_i + \rho_i \mathrm{B}_i \mathbf{G}_i - \nabla\cdot(\mathrm{B}_i p_2) + \mathbf{F}_{ji}\,,$$

$$\frac{\partial}{\partial t}(\rho_i \mathrm{B}_i E_i) = -\nabla\cdot(\rho_i \mathrm{B}_i \mathbf{U}_i E_i) - M_{ij} E_i + M_{ji} E_j + Q_{ji} - B_i Q_i\,, \quad i\,,\ j = 1\,,\ 2\,.$$

According to the second and third equations of system (1) the relation

$$\frac{\partial}{\partial t}(\rho_i \mathrm{B}_i) = -\nabla\cdot(\rho_i \mathrm{B}_i \mathbf{U}_i) - M_{ij} + M_{ji}\,, \qquad (2)$$

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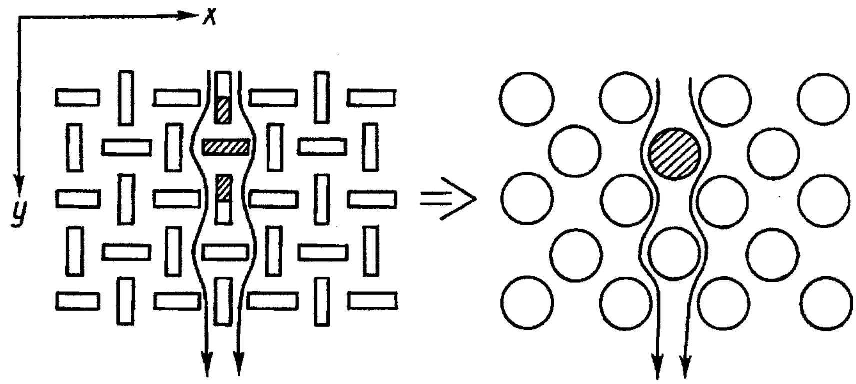

Fig. 1. Structural scheme of a bed of a filling.

is obtained, which is used to simplify the equations of motion and energy (the two last equations of system (1)):

$$\rho_i \mathrm{B}_i \frac{\partial \mathbf{U}_i}{\partial t} = - \rho_i \mathrm{B}_i \mathbf{U}_i \cdot \nabla \mathbf{U}_i + \rho_i \mathrm{B}_i \mathbf{G}_i - \nabla (\mathrm{B}_i p_2) + \mathbf{F}_{ji} ,$$

$$\rho_i B_i \frac{\partial E_i}{\partial t} = - \rho_i \mathrm{B}_i \cdot \nabla E_i + M_{ji} (E_j - E_i) + Q_{ji} - B_i Q_i . \qquad (3)$$

It should be noted that for "adequate" drying $M_{21} = 0$. The above equations are expressed in a specific form for each particular case.

Let us consider the case of steady-state drying of fruit in dryers with multirow transportation of the material by belt conveyers. The material is loaded onto the belt in a bed of thickness h, and in the transportation process it is blown from above by a heat transfer agent, which is conditioned air in this case. The technological scheme of drying requires that the equations be composed for the plane problem (x, y), where the x axis corresponds to the direction of movement of the conveyer belt and the y axis corresponds to that of motion of the heat transfer agent. For this scheme the system of basic equations (1)-(3) has the form

$$\mathrm{B}_1 + \mathrm{B}_2 = 1 , \quad \frac{\partial}{\partial x} (\rho_1 \mathrm{B}_1 U_{1x}) = - M_{12} , \quad \frac{\partial}{\partial y} (\rho_2 \mathrm{B}_2 U_{2y}) = M_{12} ,$$

$$\frac{\partial}{\partial x} [\rho_1 \mathrm{B}_1 (1 - W_1) U_{1x}] = 0 , \quad \frac{\partial}{\partial y} [\rho_2 \mathrm{B}_2 (1 - W_2) U_{2y}] = 0 ,$$

$$\rho_2 \mathrm{B}_2 U_{2y} \frac{\partial U_{2y}}{\partial y} = - \mathrm{B}_2 \frac{\partial p_2}{\partial y} - F_{21,y} - p_2 \frac{\partial \mathrm{B}_2}{\partial y} , \qquad (4)$$

$$\rho_1 \mathrm{B}_1 U_{1x} \frac{\partial E_1}{\partial x} = Q_{21} , \quad \rho_2 \mathrm{B}_2 U_{2y} \frac{\partial E_2}{\partial y} = M_{12} (E_2 - E_1) - Q_{21} .$$

Since the material to be dried is carried by the conveyer with the velocity U_{1x} with the aid of a mechanical drive, the equation of motion for the first phase is eliminated. Commercial operation of this type of dryer has shown that the aerodynamic resistance of the bed of material is low. Therefore, the equation of motion for the second phase is not important. Nevertheles, the structure of the bed should be well known, since it affects substantially the heat and mass transfer characteristics.

Before drying, fruits are cut into circular slices. In loading the cut material onto the conveyer belt, the stable position of an individual element in the filling will most likely be arbitrary. Therefore, any spatial orientation of this individual element is possible. The structure of the bed must include two limiting cases of orientation – vertical and horizontal, both of which are equiprobable. If the bed is composed of units located only in these limiting positions, the accounting for their equiprobability will result in the scheme shown in the left-hand side of Fig. 1.

It is clear from Fig. 1 that the scheme is invariant relative to the x and y axes and reflects a zigzag flow of the heat transfer agent past the units. In this case the streamlines are close to those of a flow in a spherical filling, which is studied most thoroughly. If all units of the bed are assumed to be of the same (representative) size, circular and spherical fillings will be the same, in the sense of equivalence, concerning:

a) area of contact between the filling and the heat transfer agent, if the areas $S_s^{(1)} = 4\pi R_s^2$ and $2S_c^{(1)} = 4\pi R_c^2$ are equal, which is possible at $R_s = R_c$;

b) volume of filling, if the volumes $V_s^{(1)} = 4/3\pi R_s^3$ and $2V_c^{(1)} = 2\pi R_c^2\delta_c$ are equal, which is possible when condition (a) is satisfied and the thickness of a circular slice $\delta_c = 2/3R_c$. Here R is the radius of a unit.

Equations (4) are not closed. Therefore, additional relations, determining the problem stated, are required. The kinetic equation of drying that is usually set up on the basis of experimental data in the following form is the main one:

$$\frac{dW_1}{dt} = F_1\,(W_1\,,\ T_1\,,\ \ldots)\,, \tag{5}$$

where T is the temperature. In our case for the steady-state process the left-hand side of (5) is $U_{1x}(\partial W_1/\partial x)$. With the use of the second and fourth equations of system (1), the relation between the volume density of the interphase mass transfer M_{12} and the kinetic equation of drying is found as

$$M_{12} = -\frac{\rho_1\,\mathrm{B}_1}{1 - W_1}\,F_1\,(W_1\,,\ T_1\,,\ \ldots)\,. \tag{6}$$

The density of the interphase heat flux Q_{21} is calculated as

$$Q_{21} = \alpha_{21}\,(T_2 - T_1)\,S_{12}^{(1)}\,n_1 k_{21}\,, \tag{7}$$

where the heat transfer coefficient α_{21} for a spherical filling can be determined by Drake's formula as

$$\alpha_{21} = \frac{\lambda_2}{2R_1}\,(2 + 0.46\,\mathrm{Re}_2^{0.55}\,\mathrm{Pr}_2^{0.33})\,, \quad \mathrm{Re}_2 = \frac{U_{2y}2R_1\,\rho_2}{\mu_2}\,, \quad \mathrm{Pr}_2 = \frac{\mu_2 c_2}{\lambda_2}\,. \tag{8}$$

Here $S_{12}^{(1)} = 4\pi R_1^2$; n_1 is the volume density of particles in the spherical filling:

$$n_1 = \frac{\mathrm{B}_1}{4/3\pi R_1^3}\,; \tag{9}$$

k_{21} is a correction factor for deviation of the actual conditions of the flow from those assumed in setting up formula (8). To calculate n_1, the condition of conservation of the number of particles in the solid phase can be used. The equation for the flow rate in terms of the number of particles ν_1 has the form

$$\nu_1 = n_1 S_x U_{1x} = \mathrm{const}\,, \tag{10}$$

where S_x is the cross-sectional area of the bed perpendicular to the x direction. On the other hand, the condition

$$\mathrm{B}_1 U_{1x} S_x = \nu_1\,\frac{4}{3}\,\pi R_1^3\,. \tag{11}$$

must be satisfied. Combining Eqs. (9), (10), and (11), we will find two equivalent forms of the equation for calculating the conventional radius of a solid filling:

$$R_1 = \left(\frac{3U_{1x}S_x}{4\pi\nu_1}\right)^{1/3}\mathrm{B}_1^{1/3} \quad \text{or} \quad R_1 = \left(\frac{3}{4\pi n_1}\right)^{1/3}\mathrm{B}_1^{1/3}\,. \tag{12}$$

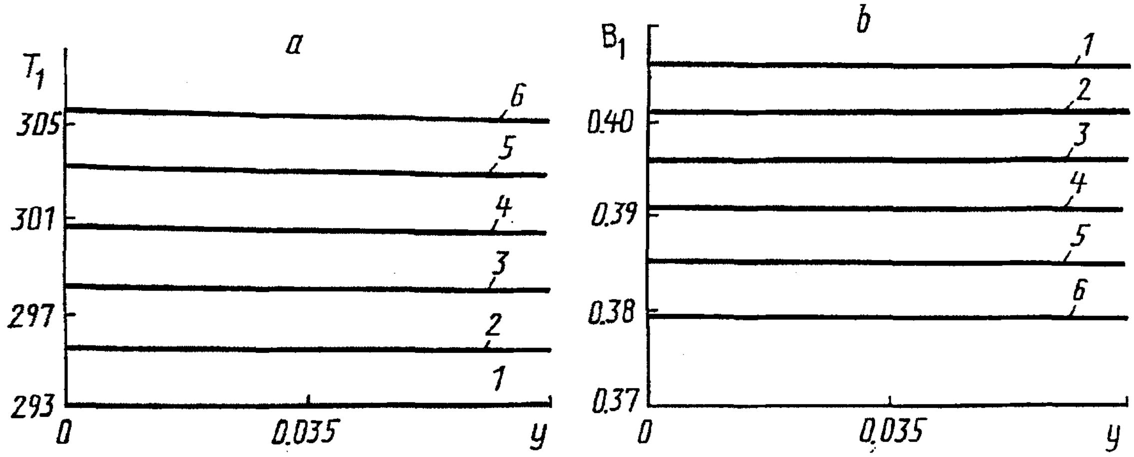

Fig. 2. Changes in the temperature T_1 (a) and volume concentration B_1 (b) of the filling over the bed thickness in cross sections located at various distances from the beginning of the belt: $x = 0$ (1), $x = 3.52$ m (2); $x = 7.04$ (3); $x = 10.56$ m (4); $x = 14.08$ m (5); $x = 17.60$ m (6). T_1, K; y, m.

As applied to the system of equations (4), the preset functions are: $\rho_1 = \rho_1(W_1)$, $\rho_2 = \rho_2(W_2, T_2)$, $c_i = c_i(W_i, T_i)$, $H_i = H_i(W_i, T_i)$, $\mu_2 = \mu_2(W_2, T_2)$, $\lambda_2 = \lambda_2(W_2, T_2)$, $U_{1x} = \text{const}$, $\nu_1 = \text{const}$, $S_x = S_x(x)$, $W_1 = W_1(x/U_{1x}, T_1)$, $Q_{21} = Q_{21}(\alpha_{21}, T_1, T_2, B_1, R_1)$.

The number of functions B_1, B_2, M_{12}, U_{2y}, W_2, E_1, E_2 to be found corresponds to the number of equations in system (4) (as was mentioned earlier, an equation of motion has been eliminated). Naturally, in this case the relation

$$dE_i \cong c_i dT_i\,, \quad i = 1\,,\,2\,. \tag{13}$$

is taken into consideration.

System of equations (4) and (5) is supplemented by initial-boundary conditions characterizing physical parameters of the material to be dried when loaded onto the conveyer belt and the heat transfer agent when entering the bed of material

$$\begin{gathered} W_1(0\,,\,y) = W_{10}(y)\,, \quad T_1(0\,,\,y) = T_{10}(y)\,, \quad B_1(0\,,\,y) = B_{10}(y)\,, \\ 0 \le y \le h\,, \quad T_2(x\,,\,0) = T_{20}(x)\,, \quad W_2(x\,,\,0) = W_{20}(x)\,, \\ u_{2y}(x\,,\,0) = U_{20}(x)\,, \quad 0 \le x \le l\,, \end{gathered} \tag{14}$$

where W_{10}, T_{10}, B_{10}, T_{20}, W_{20}, U_{20} are functions specifying the initial distribution of the physical parameters; h is the height of the bed of filling; l is the length of the conveyer belt.

Equations (4) is a system of partial differential equations of the first order, which can be expressed as a system that is solvable for the derivatives. Together with conditions (14), this system constitutes a Cauchy problem. Due to the specific properties of the physical statement of the problem the directions of movement of the components in the two-phase system are connected rigidly with the coordinate axes, and boundary conditions (14) are simultaneously initial conditions for system (4). Consequently, the spatial variables x and y are transformed into "timelike" one-sided coordinates.

This determines the structure of the algorithm for numerical solution of problem (4), (5), and (14). The algorithm is based on the possibility of treating the transformed system of equations (4) as a system that decomposes locally into a system of ordinary differential equations in the independent variables x and y that depend on the parameters y and x. For each of the equations a Cauchy problem is formulated in the corresponding variable. Solution of this problem within the descretization step is not difficult and can be performed with Euler's, Adams's,

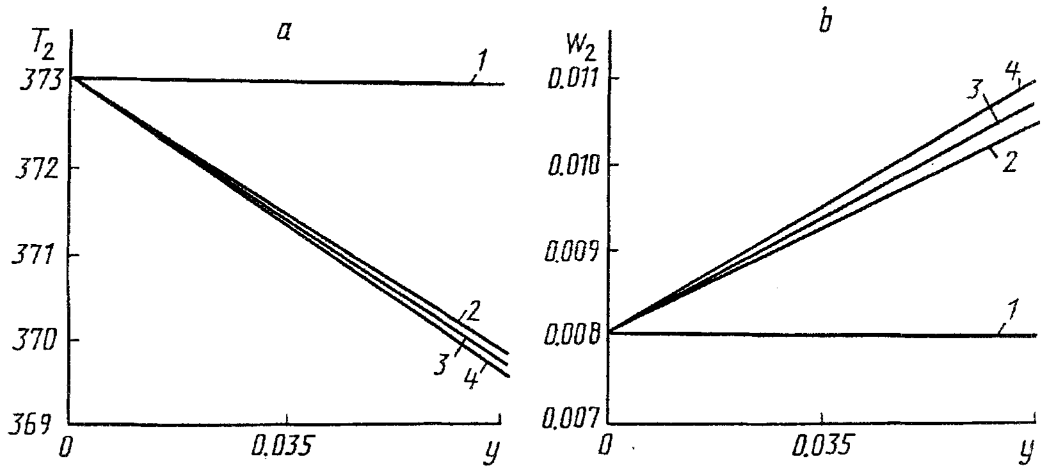

Fig. 3. Changes in the temperature T_2 of the heat transfer agent (a) and its moisture content W_2 (b) over the bed thickness in cross sections located at various distances from the beginning of the belt: $x = 0$ (1), $x = 3.52$ m (2); $x = 10.56$ m (3), $x = 17.60$ m (4).

or Runge-Kutta's method. The whole algorithm consists in successive calculation of unknown functions at nodes of drying located along straight lines parallel to an axis corresponding to a particular unknown function.

As an example, in Figs. 2 and 3 some calculated results are presented for operation of a real dryer with three-row transportation by a belt conveyer of fruits to be dried cut into circular slices at the following process parameters: $h = 0.07$ m; $l = 17.6$ m; $R_{10} = 0.04$ m; $B_{10} = 0.406$; $W_{10} = 0.87$; $\rho_{10} = 950$ kg/m^3; $T_{10} = 293$ K; $U_{1x} = 4.9 \cdot 10^{-3}$ m/sec; $T_{20} = 373$ K; $U_{20} = 0.34$ m/sec and with the following approximation of the drying curves:

$$W_1(t, T_1) = \begin{cases} W_{10} \exp\left[-kt/(a-t)\,a\right] & \text{at} \quad t \le t_p, \\ \kappa \exp(-\gamma t) & \text{at} \quad t > t_p, \end{cases}$$

where

$$t = \frac{x}{U_{1x}}; \quad t_p = 60\,[274 - 1.175\,(T_1 - 273)]; \quad k = -\ln \frac{W_p}{W_{10}} \frac{a\,(a - t_p)}{t_p};$$

$$a = t_p \ln \frac{W_{p/2}}{W_p} \left(\ln \frac{W_{p/2}^2}{W_p W_{10}} \right)^{-1}; \quad \gamma = \frac{k}{(a - t_p)^2}; \quad \kappa = \exp\left[\frac{k t_p^2}{a\,(a - t_p)} \right];$$

$$W_p = 0.002\,\frac{t_p}{2}; \quad W_{p/2} = 0.0013\,\frac{t_p}{120} + 0.62.$$

As follows from the data shown in the figures for the upper belt, a high intensity of blowing provides for an almost uniform (tending somewhat toward a decrease) distribution of the fruit temperature T_1 (Fig. 2a) and the volume concentration B_1 of the filling (Fig. 2b) throughout the thickness of the bed; as the distance from the beginning of the belt increases, T increases and B_1 decreases; the parameters T_2 and W_2 of the drying agent used on the upper belt are practically unchanged along the length of the installation (Fig. 3).

NOTATION

x, y, coordinates; t, time; p, gas pressure; E, internal energy; F, force of interphase interaction; G, volume force; Q, heat flux; M, interphase mass transfer; U, U_x, U_y, velocity and components of the velocity along x and y; W, moisture content; ρ, density; λ, thermal conductivity; μ, viscosity; c, heat capacity; B, phase indicator function (analog of the volume concentration). Subscripts: 1, material; 2, drying agent; ij, the transition $i \to j$; s, sphere; c, circle.

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